Separation Theory of an Inclined Thermal Diffusion Column with Fixed Operating Expense

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Abstract—The effect of plate spacing on the separation efficiency of inclined flat-plate thermal diffusion columns for a whole range of concentrations with a fixed operating expense has been investigated. It has been found that the maximum separation increases when the plate spacing increases. The plate spacing is generally so small that changing it, as well as changing the angle of inclination, may not cause any additional fixed charge. However, increasing the plate spacing will lead to increasing the difference of plate temperatures in order to keep the operating expense unchanged. Therefore, an additional cost is needed to maintain a high value of temperature difference.

INTRODUCTION

Thermal diffusion takes place when a temperature gradient in a mixture of two gases or liquids gives rise to a concentration gradient with one component concentrated near the hot wall and the other component concentrated near the cold wall. It was a great achievement by Clusius and Dickel[2] to introduce the thermo-gravitational thermal diffusion column that makes this separation process practical.

Although application of the thermogravitational thermal diffusion separation process has been limited by its rather high heat requirement, the process can be applied to the separation of highly valuable materials such as isotopes and rare gases which are difficult or impossible to separate by other means[18].

Recently, several improved columns[1,4~17,19,20] have been developed to increase the separation efficiency and thereby led to decreasing the heat requirements. In developing these improved columns, however, the operating expense was changed. Practically, an improved column should be developed with a fixed operating expense. It is the purpose of this work to investigate the effect of plate-spacing changes on the degree of separation of an inclined thermal diffusion column for the whole range of concentrations when the operating expense is fixed.

INCLINED FLAT-PLATE COLUMN

Clusius and Dickel showed that a horizontal temperature gradient produces not only thermal diffusion in the direction of the temperature gradient, but also a natural convection of the fluid upward near the hot surface and downward near the cold surface. These convective currents produce a cascading effect analogous to the multi-stage effect of a countercurrent extraction, and as a result a considerably greater separation may be obtained. Figure 1 illustrates the flows and fluxes in such a thermo-gravitational thermal diffusion column.

A more detailed study of the mechanism of separation in the Clusius and Dickel column indicates that the convective currents actually have two conflicting effects: the desirable cascading effect and the undesirable re-mixing effect. The convective currents have a multi-stage effect which is necessary in securing high separations, and it is an essential feature of the Clusius and Dickel column. However, since the convection brings down the fluid at the top of the column, where it is rich in one component, to the bottom of the column, where it is rich in the other component, and vice versa, there is a remixing of the two components. It appears, therefore, that proper control of the convective strength might effectively suppress this undesirable re-mixing effect while still preserving the desirable cascading effect, and thereby lead to an improved separation.

A simple and flexible way of adjusting the convective strength is to tilt a flat-plate column with hot plate on top, or to insert a wire spiral in the annular region of a concentric-tube column[12], so as to reduce the effective gravitational force. In these works, however, the condition of a fixed operating expense as well as the condition of the whole concentration was

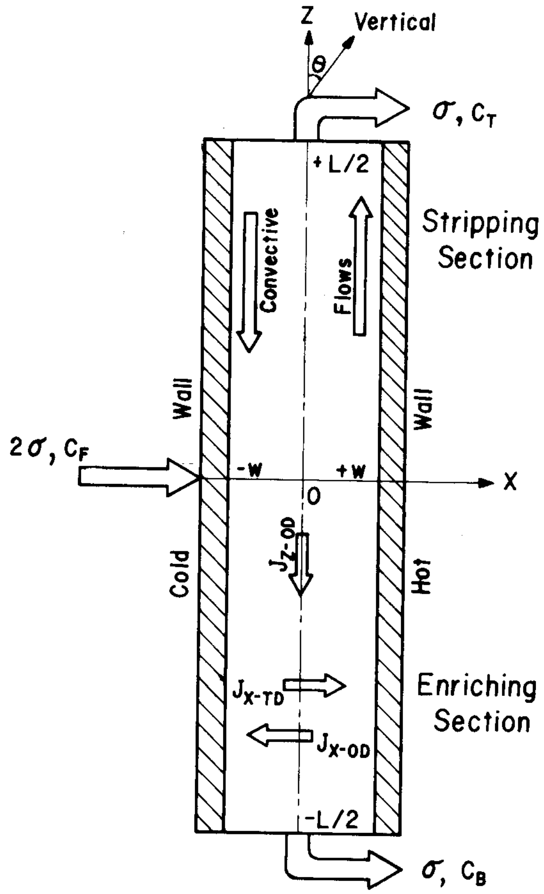

Fig. 1. Schematic diagram of a thermal diffusion column.

not taken into consideration.

THEORY OF INCLINED FLAT-PLATE COLUMN

The separation theory of thermal diffusion was first presented by Furry *et al.*[3]. Figure 1 illustrates the flows and fluxes prevailing in a continuous flow column. By integrating the transport equations, two ordinary differential equations were obtained:

$$H\left[C(1-C)-\frac{\sigma}{H}(C_T-C)\right]=(K_c+K_d)\frac{dC}{dz} \tag{1}$$

for the enriching section and

$$H\left[C(1-C)+\frac{\sigma}{H}(C_B-C)\right]=(K_c+K_d)\frac{dC}{dz} \tag{2}$$

for the stripping section. In obtaining the above equations, the product form of the concentration, $C(1-C)$, was treated as a function of z only and the flow rates were assumed the same both in the enriching and the stripping section. The system constants H, K_c and K_d are defined as

$$H=\frac{\alpha\rho\beta g\cos\theta(2\omega)^3B(\Delta T)^2}{6!\mu T} \tag{3}$$

$$K_c=\frac{\rho\beta^2g^2\cos\theta(2\omega)^7B(\Delta T)^2}{9!D\mu^2} \tag{4}$$

$$K_d=2\omega BD\rho \tag{5}$$

By setting $C(1-C)$ as some appropriate constant, viz

$$C(1-C)\cong A \tag{6}$$

the degree of separation, defined by

$$\Delta=C_T-C_B \tag{7}$$

may be obtained from Eqs. (1) and (2) associated with the boundary conditions:

$$C=C_T, \qquad \text{at } z=\frac{L}{2} \tag{8}$$

$$C=C_B, \qquad \text{at } z=\frac{-L}{2} \tag{9}$$

The solution is

$$\Delta=\frac{2AH}{\sigma}\left\{1-\exp\left[-\frac{\sigma L}{2(K_c+K_d)}\right]\right\} \tag{10}$$

Furry *et al.* obtained the simplest solution from Eq. (10) by setting $A=0.25$ applicable to the concentration range $0.3<C<0.7$:

$$\Delta=\frac{H}{2\sigma}\left\{1-\exp\left[-\frac{\sigma L}{2(K_c+K_d)}\right]\right\} \tag{11}$$

For the whole range of concentrations, the method of least squares may be carried out by finding the approppriate choice of A for the functional

$$I=\int_{C_B}^{C_T}[C(1-C)-A]^2dC \tag{12}$$

to have a minimum. In performing the integration of Eq. (12), if the upper and lower limits are substituted by Eqs. (13) and (14):

$$C_T=C_F+\frac{\Delta}{2} \tag{13}$$

$$C_B=C_F-\frac{\Delta}{2} \tag{14}$$

The result is

$$A=C_F(1-C_F)-\frac{\Delta^2}{12} \tag{15}$$

The equation of separation is then obtained by substituting Eq. (15) into Eq. (10):

$$\Delta=\left[\left(\frac{1.5}{\Delta}\right)^2+12C_F(1-C_F)\right]^{1/2}-\frac{1.5}{\Delta} \tag{16}$$

THE BEST ANGLE OF INCLINATION FOR MAXIMUM SEPARATION

Since the system constants, H, and K_c possess high power terms of ω, it plays an important role in separation efficiency in the separation equation. The plate spacing in a thermal diffusion column is generally so small that changing 2ω, as well as changing θ, will not cause any additional fixed charge. The expenditure of making a separation by thermal diffusion essentially includes two parts: a fixed charge and an operating expense. The fixed charge is roughly proportional to the equipment cost, while the operating expense is in chiefly heat. The heat transfer rate is obtainable from $kBL(\Delta T)/2\omega$. By using these terms, we shall take account of the influence of both plate spacing and inclined angle on the degree of separation when the operating expense is fixed. Therefore, Eq. (11) can be rewritten as

$$\dot{\Delta}=\frac{a(2\omega)^5\cos\theta}{2\sigma}\left\{1-\exp\frac{-\sigma L}{2[b_1(2\omega)^9\cos^2\theta+b_2(2\omega)]}\right\} \tag{17}$$

where

$$a=\frac{\alpha\rho\beta gB\left(\frac{\Delta T}{2\omega}\right)^2}{6!\mu\bar{T}}=\text{constant} \tag{18}$$

$$b_1=\frac{\rho\beta^2 g^2 B\left(\frac{\Delta T}{2\omega}\right)^2}{9!D\mu^2}=\text{constant} \tag{19}$$

$$b_2=\rho BD=\text{constant} \tag{20}$$

The best angle of inclination for a maximum separation is obtained by partially differentiating Eq. (16) with respect to θ and setting $\partial\Delta/\partial\theta=0$. After differentiation and simplification we obtain

$$e^y=1+2y-\frac{2y^2}{\sigma'} \tag{21}$$

in which

$$y=\frac{\sigma L}{2[b_1(2\omega)^9\cos^2\theta^*+b_2(2\omega)]}=\frac{\sigma'}{\frac{b_1}{b_2}(2\omega)^8\cos^2\theta^*+1} \tag{22}$$

$$\sigma'=\frac{\sigma L}{2b_2(2\omega)} \tag{23}$$

Generally, $\sigma'>100$, and the last term on the right-hand side of Eq. (21) may be neglected. Thus, $y=1.26$, and from Eq. (22)

$$\cos^2\theta^*=\frac{\frac{\sigma'}{1.26}-1}{\frac{b_1(2\omega)^8}{b_2}}\approx\frac{b_2\sigma'}{1.26(2\omega)^8 b_1} \tag{24}$$

$$\theta^*=\cos^{-1}\left[\frac{\sigma L}{2.52b_1(2\omega)^9}\right]^{1/2} \tag{25}$$

Consequently, the maximum separation for the whole range of concentrations may be obtained from Eq. (16) by the substitution of Eq. (25). The results are

$$\Delta_{max}=\left[\left(\frac{1.5}{\dot{\Delta}_{max}}\right)^2+12C_F(1-C_F)\right]^{1/2}-\frac{1.5}{\dot{\Delta}_{max}} \tag{26}$$

$$\dot{\Delta}_{max}=0.226a\left[\frac{L(2\omega)}{b_1\sigma}\right]^{1/2} \tag{27}$$

THE EFFECT OF PLATE SPACING ON THE DEGREE OF SEPARATION

Differentiating Eq. (26) with respect to (2ω), one obtains

$$\frac{\partial\Delta_{max}}{\partial(2\omega)}=\left(\frac{0.113a}{\sqrt{\frac{b_1\sigma(2\omega)}{L}}}\right)\left(\frac{1.5}{\dot{\Delta}^2_{max}}\right)\cdot\left\{1-\frac{\frac{1.5}{\dot{\Delta}_{max}}}{\left[\left(\frac{1.5}{\dot{\Delta}_{max}}\right)^2+12C_F(1-C_F)\right]^{1/2}}\right\}>0 \tag{28}$$

provided that $0<C_F<1$. Accordingly, increasing the plate spacing will increase the maximum separation with the operating expense unchanged.

A comparison of maximum separations may be made by using the experimental data of Chueh and Yeh's[1]. Materials: benzene and n-heptane; $\Delta T=164-95=69°F=38.3°C$; $2\omega=0.09cm$; $L=185\,cm$, $H/\cos\theta=0.845\,g/min$; $K_c/\cos\theta=419\,g\cdot cm/min$. If the operating expense is kept unchanged, *i.e.*, $\Delta T/(2\omega)=38.3/0.09$ ($°C/cm$), then

$$a=\frac{\frac{H}{\cos\theta}}{(2\omega)^5}=\frac{0.845}{(0.09)^5}=1.433\times10^5\,g/(cm)\,(min)$$

$$b_1=\frac{\frac{K_c}{\cos\theta}}{(2\omega)^9}=\frac{419}{(0.09)^9}=1.08\times10^{12}\,g/(cm)^8\,(min)$$

Consequently, the maximum separation and the best angle of inclination of various plate spacings are calculated from Eqs. (25) and (26), and the results are presented in Table 1.

Table 1. Comparison of separation obtained at various plate spacing

C_F, or $1-C_F$	σ (g/min)	$2\omega=0.09$ cm, $\Delta T=38.3°C$		$2\omega=0.12$ cm, $\Delta T=51.1°C$		$2\omega=0.15$ cm, $\Delta T=63.9°C$	
		θ^* (deg)	Δ_{max}	θ^* (deg)	Δ_{max}	θ^* (deg)	Δ_{max}
0.1	0.49	73.0	6.7	85.4	7.5	88.3	8.4
	0.98	65.5	4.6	83.5	5.3	87.6	5.9
	1.96	54.3	3.6	80.8	3.8	86.6	4.2
0.3	0.49	73.0	15.3	85.4	17.7	88.3	19.4
	0.98	65.5	10.6	83.5	12.4	87.6	13.8
	1.98	54.3	8.0	80.8	8.9	86.6	9.8
0.5	0.49	73.0	18.2	85.4	21.0	88.3	23.5
	0.98	65.5	12.8	83.5	14.8	87.6	16.5
	1.96	54.3	9.1	80.8	10.5	86.6	11.7

CONCLUSION

The effect of plate spacing on the separation efficiency of the inclined flat-plate thermal diffusion columns has been investigated. It has been shown both in Eq. (28) and Table 1 that the maximum separation of an inclined column increases as the plate spacing increases when the operating expense is fixed. The plate spacing in a thermal diffusion column is generally so small that changing 2ω, as well as changing θ, will not cause any additional fixed charge. However, increasing 2ω will lead to an increasing ΔT in order to maintain the $\Delta T/2\omega$ constant, and therefore, some additional cost is needed to maintain the higher ΔT. For the example given, it is evident in Table 1 that ΔT must be kept as high as 63.9°C when the plate spacing is 0.15 cm. Since the boiling points of both benzene and n-heptane are about 80°C, the temperature of the cold plate of a thermal diffusion must be held as low as 0°C. In this case, although the heat transfer rate remains the same, kBL (38.3/0.09), some additional expense is needed to keep the cold plate as low as 0°C.

NOMENCLATURE

A	constant defined by Eq. (6) or Eq. (15)
a	system constant defined by Eq. (18), ($g\ cm^{-1}\ s^{-1}$)
B	column width, (cm)
b_1	system constant defined by Eq. (19), ($g\ cm^{-8}\ s^{-1}$)
b_2	system constant defined by Eq. (20), ($g\ s^{-1}$)
c	fractional mass concentration of component 1
C_B, C_T	C in the product stream existing from the stripping, enriching section
C_F	C in the feed stream
D	ordinary diffusion coefficient, ($cm^2\ s^{-1}$)
g	gravitational acceleration, ($cm\ s^{-2}$)
H	system constant defined by Eq. (3), ($g\ s^{-1}\ cm^{-1}$)
J_{x-OD}, J_{x-TD}	mass flux of component 1 in x-direction due to ordinary, thermal diffusion, ($g\ cm^{-2}\ s^{-1}$)
J_{z-OD}	mass flux of component 1 in z-direction due to drdinary diffusion, ($g\ cm^{-2}\ s^{-1}$)
K_c	system constant defined by Eq. (4), ($g\ s^{-1}$)
K_d	system constant defined by Eq. (5), ($g\ s^{-1}$)
k	thermal conductivity of fluid, (watt $cm^{-2}\ K^{-1}$)
L	column length, (cm)
T	absolute temperature, (K)
$\bar{T}$	reference temperature, (K)
ΔT	difference in temperature of hot and cold surfaces, (K)
x	axis normal to the plate surfaces, (cm)
y	value determined from Eq. (21)
z	axis parallel to the flowing direction, (cm)

Greek Symbols

α	thermal diffusion constant
β	$-(\partial\rho/\partial T)_{\bar{T}}$, ($g\ cm^{-3}\ K^{-1}$)
Δ	C_T-C_B
$\dot{\Delta}$	Δ obtained when $0.3<C<0.7$
Δ_{max}, $\dot{\Delta}_{max}$	maximum value of Δ, $\dot{\Delta}$
μ	viscosity of fluid, ($g\ cm^{-1}\ s^{-1}$)
θ	angle of inclination of column plate from the vertical, (deg)
θ^*	optimal θ for maximum separation, (deg)
ρ	mass density, ($g\ cm^{-3}$)
σ	mass flow-rate, ($g\ s^{-1}$)
σ'	$\sigma L/2b_2(2\omega)$
ω	one-half of the plate spacing of the column, (cm)

REFERENCES

1. Chueh, P.L. and H.M. Yeh, *AIChE J.*, **13**, 37 (1967).
2. Clusius, K. and G. Dickel, *Naturwiss*, **26**, 546(L) (1938).
3. Furry, W.H., R.C. Jones and L. Onsager, *Phys. Rev.*, **55**, 1083 (1939).
4. Lorenz, M. and A.H. Emery, Jr., *Chem. Eng. Sci.*, **11**, 16 (159).
5. Powers, J.E. and C.R. Wilke, *AIChE J.*, **3**, 213 (1957).
6. Ramser, J.H., *Ind. Eng. Chem.*, **49**, 155 (1957).
7. Sullivan, L.J., T.C. Ruppel and C.B. Willingham,

Ind. Eng. Chem., **47**, 110 (1957).
8. Tsai, S.W. and H.M. Yeh, *Canadian J. of Chem. Eng.*, **63**, 406 (1985).
9. Tsai, S.W. and H.W. Yeh, *J. ChE, Japan*, **19**(6), 548 (1986).
10. Tsai, S.W. and H.M. Yeh, *I&EC Fundamentals*, **25**, 367 (1986).
11. Washall, T.A. and F.W. Molpolder, *I&EC Process Des. Dev.*, **1**, 26 (1962).
12. Yeh, H.M. and H.C. Ward, *Chem. Eng. Sci.*, **26**, 937 (1971).
13. Yeh, H.M. and C.S. Tsai, *ibid.*, **27**, 2065 (1972).
14. Yeh, H.M. and S.M. Cheng, *ibid.*, **28**, 1083 (1973).
15. Yeh, H.M. and T.Y. Chu, *ibid.*, **29**, 1421 (1974).
16. Yeh, H.M. and F.K. Ho, *ibid.*, **30**, 1381 (1975).
17. Yeh, H.M. and S.W. Tsai, *J. ChE Japan*, **14**(2), 90 (1981).
18. Yeh, H.M. and S.C. Yang, *Chem. Eng. Sci.*, **39**, 1277 (1984).
19. Yeh, H.M., S.W. Tsai and C.S. Lin, *AIChE J.*, **32**(5), 971 (1986).
20. Yeh, H.M., S.W. Tsai and W.H. Chen, *Canadian J. of Chem. Eng.*, **64**, 687 (1986).

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